Sum-Rate Capacity of Correlated Multi-User MIMO Channels

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Abstract— **This paper investigates the sum-rate capacity of a multi-user broadcast channel (BC) assuming that users have different transmit correlation matrices. The sum-rate capacity of two-user BC has recently been investigated. As an extension of the two-user BC case, we analyze the sum-rate capacity of a multi-user BC. It is shown by means of upper-bound analysis that the sum-rate capacity of a correlated multi-user BC is maximized when the phases of the transmit correlation coefficient of users are maximally scattered, which implies that the principal eigenvectors of the transmit correlation matrix of users are orthogonal to each other. Finally, the analytic results are verified by computer simulation.**

I. Introduction

The use of multiple antennas (so-called MIMO) in wireless systems has attracted much attention due to its potential for significant increase of the capacity [1]–[3]. Especially, multi-user broadcast channel (BC) can dramatically increase the sum-rate capacity by using multiple transmit antennas, even with the use of a single receive antenna [3]–[5]. A number of research works have devoted to the analysis of the capacity of multi-user BC, mostly assuming that the channels are independent and identically distributed (i.i.d.) [4], [5]. However, recent measurements have shown that multi-antenna BC is often correlated in practice [6], [7].

The effect of transmit correlation on the sum-rate capacity of a multi-user BC was investigated assuming that users have the same transmit correlation matrices [8]. However, users may have different transmit correlation characteristics due to different distance from the BS, angle of departure, angle of arrival, and Doppler spread [9], [10]. Recently, the effect of the transmit correlation on the sum-capacity of a two-user BC has been analyzed assuming that two users have different transmit correlation matrices [11]. The sum-rate capacity of a correlated two-user BC can be represented in terms of the transmit correlation matrix of three users and it is maximized when the principal eigenvectors of the transmit correlation matrix of two users are orthogonal to each other.

In this paper, as an extension of [11], we investigate the sum-rate capacity of a correlated multi-user BC assuming that each user experiences different transmit correlation. We consider a three-user case where the BS has three transmit antennas and simultaneously transmits signals to three users each of which has a single receive antenna. For ease of mathematical tractability, we assume the use of equal power allocation in high signal-to-noise ratio (SNR) environments [4], [12]. Our major findings inlude: The sum-rate capacity of a correlated three-user BC is represented in terms of the transmit correlation matrix of three users in a closed form. It is maximized when the phases of the transmit correlation coefficient of three users are maximally scattered, which geometrically implies that the principal eigenvectors of the transmit correlation matrix of three users are orthogonal to each other in the presence of high transmit correlation. Finally, it is shown that the analytic result can be extended to multi-user cases.

The remainder of this paper is organized as follows. Section II describes a correlated three-user BC model in consideration. Section III analyzes the sum-rate capacity of a correlated three-user BC. Section IV verifies the analytic results by computer simulation. Finally, conclusions are given in Section V.

II. Correlated Three-User Broadcast Channel

We consider a three-user BC as illustrated in Fig. 1, where the BS has three transmit antennas and each user has a single receive antenna. Let $\mathbf{x} = [x_1 \;\; x_2 \;\; x_3]^T$ be the transmitted signal vector from the BS and $\mathbf{h}_m = [h_{m1} \;\; h_{m2} \;\; h_{m3}]$ be the channel vector from the BS to user m for $m = 1, 2, 3$, where the superscript T denotes transpose. Then, the received signal of user m can be represented as [3]

$$y_m = \mathbf{h}_m \mathbf{x} + n_m \tag{1}$$

where n_m denotes additive noise of user m, which is assumed to be a zero-mean complex white Gaussian random variable with variance N_0.

Assuming that the channel vector $\mathbf{h}_m$ has transmit correlation represented by a transmit correlation matrix $\mathbf{R}_m$, the channel vector can be represented as [11]

$$\mathbf{h}_m = \tilde{\mathbf{h}}_m \mathbf{R}_m^{1/2} \tag{2}$$

where the elements of $\tilde{\mathbf{h}}_m = [\tilde{h}_{m1} \;\; \tilde{h}_{m2} \;\; \tilde{h}_{m3}]$ are i.i.d. zero-mean complex Gaussian random variables with unit-variance and $\mathbf{R}_m^{1/2}$ denotes the square root of $\mathbf{R}_m$ (i.e., $\mathbf{R}_m = \mathbf{R}_m^{1/2}\mathbf{R}_m^{*/2}$). Here, the superscript $*$ denotes conjugate transpose. Assuming that the channel has exponentially decaying correlation, the transmit correlation matrix of user m can be

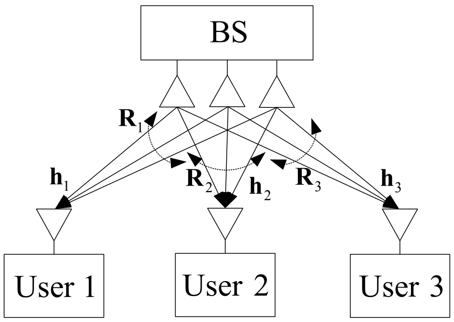

Fig. 1. Modeling of a correlated three-user BC.

represented as [13], [14]

$$\mathbf{R}_m = \begin{bmatrix} 1 & \rho_m & \rho_m^2 \\ \rho_m^* & 1 & \rho_m \\ \rho_m^{2*} & \rho_m^* & 1 \end{bmatrix} \quad (3)$$

where $\rho_m (= \alpha_m e^{j\theta_m})$ is the transmit correlation coefficient of user m. Here, $\alpha_m (0 \leq \alpha_m < 1)$ and $\theta_m (-\pi \leq \theta_m < \pi)$ denote its amplitude and phase, respectively, and $j = \sqrt{-1}$.

III. Capacity Analysis of Correlated Three-User Broadcast Channels

Assume that the BS has perfect channel information of three users and each user has also perfect information of its own channel, and that the SNR is high. Then, the achievable sum-rate of a three-user BC with the use of equal-power allocation can be represented as [3]

$$C = E\left\{\log_2 \det\left(\mathbf{I} + \frac{\gamma}{3}\mathbf{H}^*\mathbf{H}\right)\right\} \quad (4)$$

where $E\{\cdot\}$ denotes the expectation, $\mathbf{I}$ denotes a (3×3) identity matrix, $\gamma = P/N_0$ represents the SNR, P denotes the total transmit power, $\det(\cdot)$ denotes the determinant of a matrix, and $\mathbf{H} (= [\mathbf{h}_1^T \ \mathbf{h}_2^T \ \mathbf{h}_3^T]^{\mathrm{T}})$ denotes the channel matrix of three users. Note that (4) can be referred to the sum-rate capacity since equal-power allocation is optimum in high SNR condition [4], [12].

A. Sum-Rate Capacity of Correlated Three-User BC

We study the sum-rate capacity of a correlated three-user BC by means of upper-bound analysis. The following theorem presents the main result of this subsection.

Theorem 1: The sum-rate capacity of a correlated three-user BC is upper-bounded by

$$C \leq \log_2 \begin{bmatrix} (1+\gamma)^3 \\ -\frac{1}{9}\left\{\begin{array}{l} \mathrm{tr}(\mathbf{R}_1\mathbf{R}_2) \\ +\mathrm{tr}(\mathbf{R}_2\mathbf{R}_3) + \mathrm{tr}(\mathbf{R}_3\mathbf{R}_1) \end{array}\right\}\gamma^2(1+\gamma) \\ +\frac{1}{27}\{\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3) + \mathrm{tr}(\mathbf{R}_3\mathbf{R}_2\mathbf{R}_1)\}\gamma^3 \end{bmatrix} \quad (5)$$

where $\mathrm{tr}(\ \cdot\ )$ denotes the trace of a matrix.

Proof: From the property $\det(\mathbf{I} + \mathbf{XY}) = \det(\mathbf{I} + \mathbf{YX})$ [1], (4) can be rewritten as

$$C = E\left\{\log_2 \det\left(\mathbf{I} + \frac{\gamma}{3}\mathbf{H}\mathbf{H}^*\right)\right\}. \quad (6)$$

Denoting $\mathbf{H} = [\mathbf{h}_1^T \ \mathbf{h}_2^T \ \mathbf{h}_3^T]^T$, (6) can be represented as

$$C = E\left\{\log_2 \det\left(\begin{bmatrix} 1 + \frac{\gamma\mathbf{h}_1\mathbf{h}_1^*}{3} & \frac{\gamma\mathbf{h}_1\mathbf{h}_2^*}{3} & \frac{\gamma\mathbf{h}_1\mathbf{h}_3^*}{3} \\ \frac{\gamma\mathbf{h}_2\mathbf{h}_1^*}{3} & 1 + \frac{\gamma\mathbf{h}_2\mathbf{h}_2^*}{3} & \frac{\gamma\mathbf{h}_2\mathbf{h}_3^*}{3} \\ \frac{\gamma\mathbf{h}_3\mathbf{h}_1^*}{3} & \frac{\gamma\mathbf{h}_3\mathbf{h}_2^*}{3} & 1 + \frac{\gamma\mathbf{h}_3\mathbf{h}_3^*}{3} \end{bmatrix}\right)\right\}. \quad (7)$$

Using Jensen's inequality, (7) can be represented as

$$C \leq \log_2 \begin{bmatrix} E\{1 + \gamma\mathbf{h}_1\mathbf{h}_1^*/3\}E\{1 + \gamma\mathbf{h}_2\mathbf{h}_2^*/3\} \\ \times E\{1 + \gamma\mathbf{h}_3\mathbf{h}_3^*/3\} \\ -E\{1 + \gamma\mathbf{h}_1\mathbf{h}_1^*/3\}E\{\gamma^2\mathbf{h}_2\mathbf{h}_3^*\mathbf{h}_3\mathbf{h}_2^*/9\} \\ -E\{1 + \gamma\mathbf{h}_2\mathbf{h}_2^*/3\}E\{\gamma^2\mathbf{h}_3\mathbf{h}_1^*\mathbf{h}_1\mathbf{h}_3^*/9\} \\ -E\{1 + \gamma\mathbf{h}_3\mathbf{h}_3^*/3\}E\{\gamma^2\mathbf{h}_1\mathbf{h}_2^*\mathbf{h}_2\mathbf{h}_1^*/9\} \\ +E\{\gamma^3\mathbf{h}_1\mathbf{h}_2^*\mathbf{h}_2\mathbf{h}_3^*\mathbf{h}_3\mathbf{h}_1^*/27\} \\ +E\{\gamma^3\mathbf{h}_3\mathbf{h}_2^*\mathbf{h}_2\mathbf{h}_1^*\mathbf{h}_1\mathbf{h}_3^*/27\} \end{bmatrix}. \quad (8)$$

It can be shown from $\mathbf{h}_m = \tilde{\mathbf{h}}_m\mathbf{R}_m^{1/2}$, $E\{\tilde{\mathbf{h}}_m^*\mathbf{R}\tilde{\mathbf{h}}_m\} = \mathrm{tr}(\mathbf{R})$, $\mathrm{tr}(\mathbf{R}_m) = 3$ for $m = 1,\ 2,\ 3$, $E\{\tilde{\mathbf{h}}_1^*\mathbf{X}\tilde{\mathbf{h}}_2\tilde{\mathbf{h}}_2^*\mathbf{Y}\tilde{\mathbf{h}}_1\} = \mathrm{tr}(\mathbf{XY})$, and $E\{\tilde{\mathbf{h}}_1^*\mathbf{X}\tilde{\mathbf{h}}_2\tilde{\mathbf{h}}_2^*\mathbf{Y}\tilde{\mathbf{h}}_3\tilde{\mathbf{h}}_2^*\mathbf{Z}\tilde{\mathbf{h}}_3\} = \mathrm{tr}(\mathbf{XYZ})$ that (8) can be rewritten as

$$C \leq \log_2 \begin{bmatrix} (1+\gamma)^3 \\ -\frac{1}{9}\left\{\begin{array}{l} \mathrm{tr}(\mathbf{R}_1\mathbf{R}_2) \\ +\mathrm{tr}(\mathbf{R}_2\mathbf{R}_3) + \mathrm{tr}(\mathbf{R}_3\mathbf{R}_1) \end{array}\right\}\gamma^2(1+\gamma) \\ +\frac{1}{27}\{\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3) + \mathrm{tr}(\mathbf{R}_3\mathbf{R}_2\mathbf{R}_1)\}\gamma^3 \end{bmatrix}.$$

This completes the proof of the theorem. ■

B. Capacity Maximizing Condition

In this subsection, we investigate the condition that maximizes the sum-rate capacity of a correlated three-user BC. The following theorem presents the main result of this subsection.

Theorem 2: The sum-rate capacity of a three-user BC is maximized when the phase difference among the transmit correlation coefficient of three users is $2\pi/3$ in the presence of high transmit correlation.

Proof: It can be shown from (3) that

$$\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2) = 3 + 4\cos(\theta_1 - \theta_2) + 2\cos 2(\theta_1 - \theta_2) \quad (9)$$

and

$$\begin{aligned} \mathrm{tr}(\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3) = 3 &+ 4\begin{pmatrix} \cos(\theta_1 - \theta_2) \\ + \cos(\theta_2 - \theta_3) + \cos(\theta_3 - \theta_1) \end{pmatrix} \\ &+2\begin{pmatrix} \cos 2(\theta_1 - \theta_2) \\ + \cos 2(\theta_2 - \theta_3) + \cos 2(\theta_3 - \theta_1) \end{pmatrix} \\ &+2\begin{pmatrix} \cos(\theta_1 + \theta_2 - 2\theta_3) \\ + \cos(\theta_2 + \theta_3 - 2\theta_1) \\ + \cos(\theta_3 + \theta_1 - 2\theta_2) \end{pmatrix} \end{aligned} \quad (10)$$

in the presence of high transmit correlation (i.e., $\alpha_m \to 1$ for $m = 1,\ 2,\ 3$). Letting $\omega_1 = \theta_1 - \theta_2$ and $\omega_2 = \theta_2 - \theta_3$, (9) and (10) can be rewritten as

$$\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2) = 3 + 4\cos\omega_1 + 2\cos 2\omega_1 \quad (11)$$

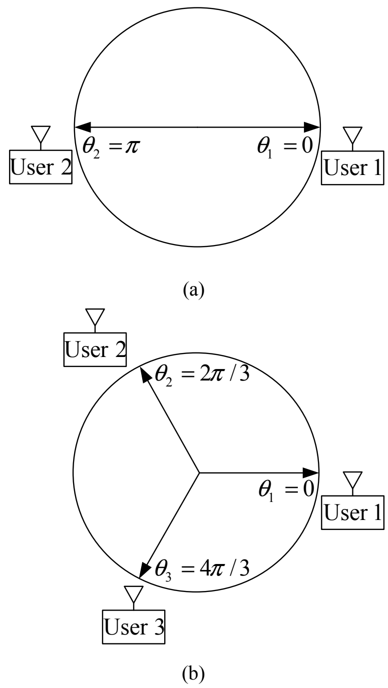

Fig. 2. Sum-rate capacity maximizing condition. (a) Two-user BC. (b) Three-user BC.

and

$$\begin{aligned} \operatorname{tr}(\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3) = 3 &+ 4\left(\cos\omega_1 + \cos\omega_2 + \cos(\omega_1+\omega_2)\right) \\ &+2\left(\cos 2\omega_1 + \cos 2\omega_2 + \cos 2(\omega_1+\omega_2)\right) \\ &+2\begin{pmatrix}\cos(\omega_1 + 2\omega_2) \\ +\cos(2\omega_1+\omega_2) + \cos(\omega_1-\omega_2)\end{pmatrix}. \end{aligned} \tag{12}$$

Letting

$$\begin{aligned} p = 4(&\cos\omega_1 + \cos\omega_2 + \cos(\omega_1+\omega_2)) \\ &+2(\cos 2\omega_1 + \cos 2\omega_2 + \cos 2(\omega_1+\omega_2)) \end{aligned} \tag{13}$$

and

$$q = 2(\cos(2\omega_1+\omega_2) + \cos(\omega_1-\omega_2) + \cos(\omega_1+2\omega_2)), \tag{14}$$

(5) can be rewritten as

$$C \le \log_2\left[1 + 3\gamma + \frac{1}{9}(18-p)\gamma^2 + \frac{1}{27}(6-p+2q)\gamma^3\right]. \tag{15}$$

The condition maximizing (15) is equivalent to the maximizing condition of $f = -p + 2q$ in high SNR environments. Since $\nabla f = 0$, $\partial^2 f(\omega_1, \omega_2)/\partial^2\omega_1 < 0$ and $\det \nabla^2 f > 0$ for $\omega_1 = \omega_2 = \pm 2\pi/3$, it can be shown that f is maximized when

$$|\theta_1 - \theta_2| = |\theta_2 - \theta_3| = |\theta_3 - \theta_1| = 2\pi/3, \tag{16}$$

which also maximizes (5). This completes the proof of the theorem. ■

Theorem 2 means that the sum-rate capacity of a correlated three-user BC is maximized when the phases of the transmit correlation coefficients of three users are maximally scattered in $[0, 2\pi)$. It can be conjectured from the two- and three-user cases as illustrated in Fig. 2 that the sum-rate capacity of a correlated M-user BC is maximized when the phases of the transmit correlation coefficient of M users are maximally scattered in $[0, 2\pi)$ (i.e., separated by $2\pi/M$).

C. Geometrical Interpretation

In this subsection, we investigate the geometrical meaning of the sum-rate capacity condition of a correlated three-user BC. The following theorem presents the main result of this subsection.

Theorem 3: The sum-rate capacity of a three-user BC is maximized when the principal eigenvectors of the transmit correlation matrix of three users are orthogonal to each other in the presence of high transmit correlation.

Proof: By means of eigen-value decomposition, it can be shown that the principal eigenvector of $\mathbf{R}_m$ can be represented as

$$\mathbf{u}_{m,\max} = [1 \;\; e^{-j\theta_m} \;\; e^{-j2\theta_m}] \tag{17}$$

when the transmit correlation is high (i.e., $\alpha_m \to 1$ for $m = 1, 2, 3$). From [15] and (17), it can be shown that

$$\cos|\phi_1 - \phi_2| = \frac{|\mathbf{u}^*_{1,\max}\mathbf{u}_{2,\max}|}{||\mathbf{u}^*_{1,\max}||\,||\mathbf{u}_{2,\max}||} \tag{18}$$

$$= \frac{\sqrt{3 + 4\cos|\theta_1-\theta_2| + 2\cos 2|\theta_1-\theta_2|}}{(2\cos\theta_1 + 1)(2\cos\theta_2+1)} \tag{19}$$

where ϕ_m represents the direction of $\mathbf{u}_{m,\max}$, $|\mathbf{u}^*_{1,\max}\mathbf{u}_{2,\max}|$ denotes the inner product of $\mathbf{u}^*_{1,\max}$ and $\mathbf{u}_{2,\max}$, and $||\cdot||$ denotes the Euclidean norm of a vector. Thus, we have

$$|\phi_1 - \phi_2| = \cos^{-1}\left(\frac{\sqrt{3 + 4\cos|\theta_1-\theta_2| + 2\cos 2|\theta_1-\theta_2|}}{(2\cos\theta_1 + 1)(2\cos\theta_2+1)}\right). \tag{20}$$

Substituting the condition $|\theta_1 - \theta_2| = 2\pi/3$ in (16) into (19), we have $|\phi_1 - \phi_2| = \pi/2$. Similarly, $|\theta_2 - \theta_3| = 2\pi/3$ and $|\theta_3 - \theta_1| = 2\pi/3$ in (16) imply $|\phi_2 - \phi_3| = \pi/2$ and $|\phi_3 - \phi_1| = \pi/2$, respectively. Thus, the condition that maximizes the sum-rate capacity in Theorem 2 geometrically implies that

$$|\phi_1 - \phi_2| = |\phi_2 - \phi_3| = |\phi_3 - \phi_1| = \pi/2.$$

This completes the proof of the theorem. ■

Theorem 3 shows that the condition (16) implies that the principal eigenvectors of the transmit correlation matrix of three users are orthogonal to each other. This is an extension of capacity maximizing condition of a correlated two-user BC to the three-user case, as illustrated in Fig. 3.

IV. Simulation Results

The analytic results are verified by computer simulation. Fig. 4 depicts the sum-rate capacity of a correlated three-user BC according to the phase difference between the transmit correlation coefficients when the SNR is 10 and 20 dB. It can be seen that the sum-rate capacity increases as the

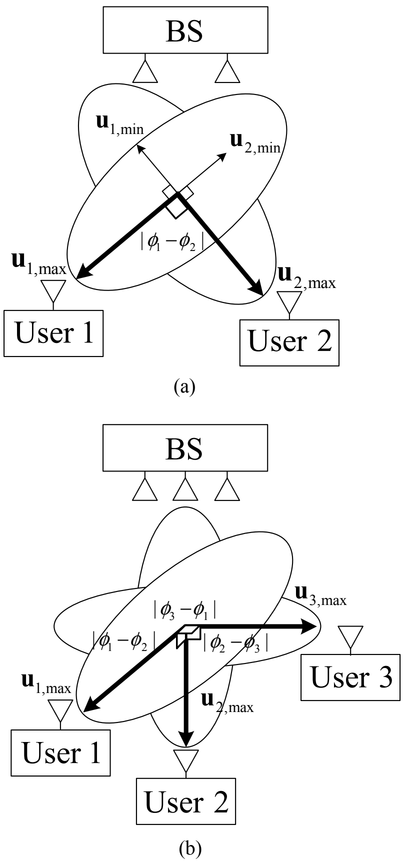

Fig. 3. Geometrical meaning of sum-rate capacity maximizing condition. (a) Two-user BC. (b) Three-user BC.

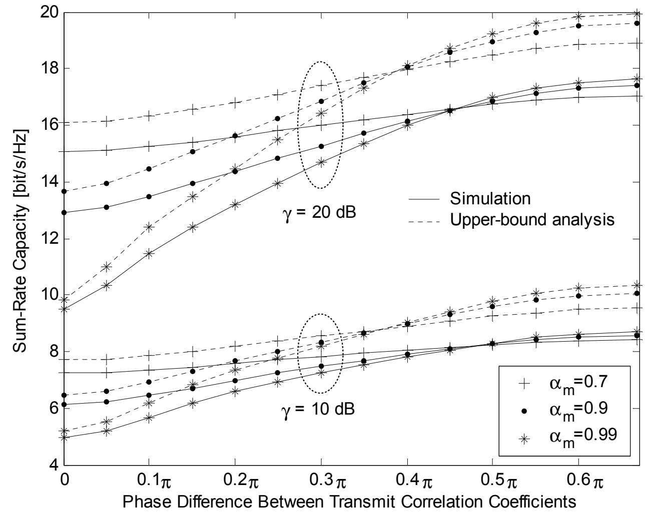

Fig. 4. Sum-rate capacity of a correlated three-user BC according to the phase difference between the transmit correlation coefficients.

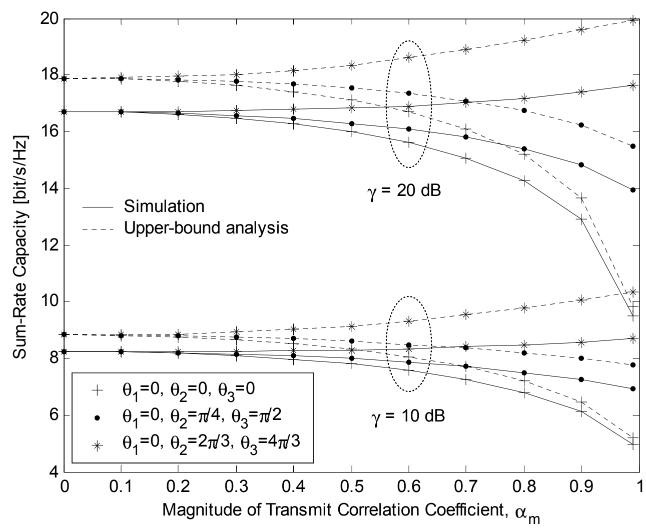

Fig. 5. Sum-rate capacity of a correlated three-user BC according to the magnitude of the transmit correlation coefficient.

phase difference between the transmit correlation coefficients increases, and thus it is maximized when $|\theta_1-\theta_2| = |\theta_2-\theta_3| = |\theta_3-\theta_1| = 2\pi/3$ as shown by Theorem 2. It can also be seen that the analytic upper bound is somewhat loose compared to the simulation results, but it still faithfully reflects the effect of transmit correlation on the sum-rate capacity.

Fig. 5 depicts the sum-rate capacity of a correlated three-user BC according to the magnitude of transmit correlation. It can be seen that the sum-rate capacity of a correlated three-user BC is maximized when $|\theta_1-\theta_2| = |\theta_2-\theta_3| = |\theta_3-\theta_1| = 2\pi/3$. It can also be seen that the sum-rate capacity gap between the maximum and the minimum condition increases as the magnitude of transmit correlation increases. It is important to note that as the magnitudes of the transmit correlation coefficient increase, the sum-rate capacity of a correlated three-user BC somewhat increases in this maximum condition, which is similar to the tendency in a correlated two-user BC.

V. Conclusion

In this paper, we have analyzed the sum-rate capacity of a multi-user BC with an assumption that users have different transmit correlation matrices. We have represented the sum-rate capacity of a correlated three-user BC can be represented in a closed form. Then, we have shown that it is maximized when the phases of the transmit correlation coefficient are maximally scattered. We have also shown that the condition maximizing the sum-rate capacity is geometrically equal to when the principal eigenvectors of the transmit correlation matrix of three users are orthogonal to each other.

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