MIMO Gaussian Broadcast Channels with Common Messages

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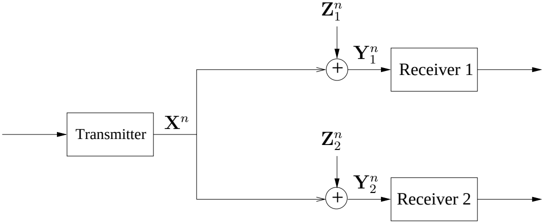
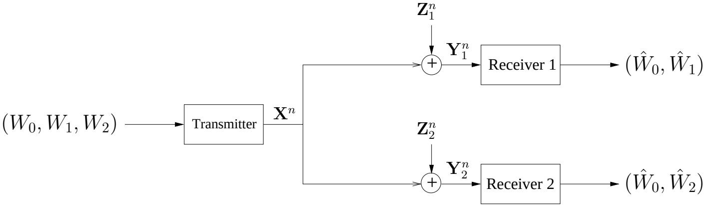
Joint work with Ruoheng Liu, Vincent Poor and Shlomo Shamai

Aligned MIMO Gaussian Broadcast Channel



- $\mathbf{Z}_k[m]$ are i.i.d. $\mathcal{N}(0, \mathbf{N}_k)$
- Matrix constraint: $\frac{1}{n}\sum_{m=1}^{n}(\mathbf{X}[m]\mathbf{X}[m]^t) \preceq \mathbf{S}$
- A canonical model for MIMO Gaussian broadcast channel

Common Message Problem



- A natural scheme: Superposition + dirty-paper coding (Jindal-Goldsmith '04)
- Is the natural scheme optimal?

State of the Art

- The natural scheme is optimal when considering:
  - Private messages
    (Weingarten-Steinberg-Shamai '04)
  - Common and one private messages
    (Weingarten-Steinberg-Shamai '06)
  - Common vs. sum-private rates
    (Weingarten, Ph.D. thesis)

- Full capacity region remains unknown

Proof Techniques

- Private messages:
  - Enhancement + entropy power inequality
- Degraded message sets:
  - Common message relaxation
- Common vs. sum-private rates:
  - Enhancement + extremal inequality

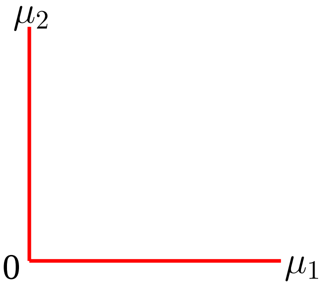
An Extremal Inequality

Hanan Weingarten, Tie Liu, Shlomo Shamai (Shitz), Yossef Steinberg, and Pramod Viswanath, "The capacity region of the degraded multiple-input multiple-output compound broadcast channel," *IEEE Transactions on Information Theory*, vol. 55, no. 11, pp. 5011–5023, November 2009.

This Talk

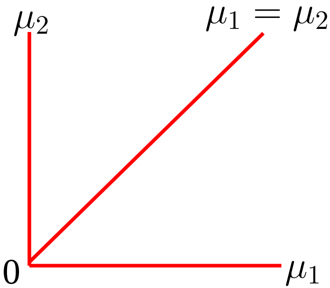
$$\max\ [R_0 + \mu_1 R_1 + \mu_2 R_2] \quad \text{for}\ \ \mu_1, \mu_2 \geq 0$$

Known Results



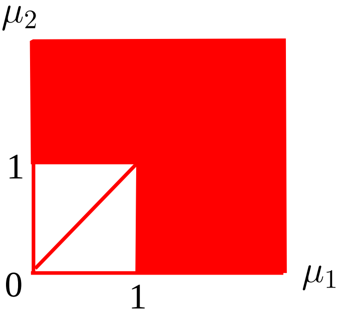
- $\mu_1\mu_2 = 0$: Degraded message sets

Known Results



- $\mu_1\mu_2 = 0$: Degraded message sets
- $\mu_1 = \mu_2$: Common vs. sum-private rates

Another Simple Case: $\mu_1 \geq 1$ or $\mu_2 \geq 1$

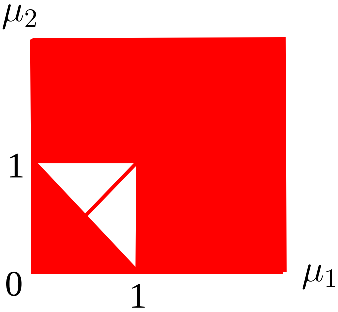


- The converse ($\mu_1 \geq 1$):
  - $R_0 + \mu_1 R_1 + \mu_2 R_2 \leq \mu_1 (R_0 + R_1) + \mu_2 R_2$
  - $(R_0 + R_1, R_2) \in \mathcal{C}_{Priv}(\mathbf{S}, \mathbf{N}_1, \mathbf{N}_2)$
- Achievability: $R_0 = 0$

Main Result

The natural scheme achieves the weighted sum capacity for

$$\mu_1 + \mu_2 \leq 1$$



Caveats

- Full capacity region remains unknown
  - That $\mu_1, \mu_2 \leq 1$ but $\mu_1 + \mu_2 \geq 1$ corresponds to a significant portion of the achievable rate region
- An earlier result (Weingarten-Steinberg-Shamai '06):
  - The natural scheme achieves the capacity region when

$$R_0 \geq \min_{k=1,2} \left[\frac{1}{2} \log \frac{|\mathbf{S} + \mathbf{N}_k|}{|\mathbf{N}_k|(1 + \xi_k)} \right]$$

    where ξ_k is the smallest eigenvalue of $\mathbf{N}_k^{-1/2} \mathbf{S} \mathbf{N}_k^{-1/2}$
  - Does not extend to average total power constraint

A Single-Letter Outer Bound to the Capacity Region

- For any achievable rate triple (R_0, R_1, R_2),

$$\begin{aligned} R_0 &\leq \min[I(U;\mathbf{Y}_1), I(U;\mathbf{Y}_2)] \\ R_0 + R_2 &\leq \min[I(U;\mathbf{Y}_1), I(U;\mathbf{Y}_2)] + I(V;\mathbf{Y}_2|U) \\ R_0 + R_1 + R_2 &\leq \min[I(U;\mathbf{Y}_1), I(U;\mathbf{Y}_2)] + \\ & \quad I(\mathbf{X};\mathbf{Y}_1|U,V) + I(V;\mathbf{Y}_2|U) \end{aligned}$$

  for some $p(u, v, \mathbf{x}, \mathbf{y}_1, \mathbf{y}_2) = p(u, v)p(\mathbf{x}|u, v)p(\mathbf{y}_1, \mathbf{y}_2|\mathbf{x})$

- A weakened version of the “New Jersey” bound (Liang-Kramer-Shamai ’08)

A Single-Letter Outer Bound to the Weighted Sum Capacity

For any $0 \le \mu_1 \le \mu_2 \le 1$,

$$\begin{aligned} & R_0 + \mu_1 R_1 + \mu_2 R_2 \\ & \qquad = \mu_1(R_0 + R_1 + R_2) + (\mu_2 - \mu_1)(R_0 + R_2) + (1 - \mu_2)R_0 \\ & \qquad \le \min[I(U;\mathbf{Y}_1), I(U;\mathbf{Y}_2)] + \mu_1 I(\mathbf{X};\mathbf{Y}_1|U,V) + \mu_2 I(V;\mathbf{Y}_2|U) \end{aligned}$$

for some $p(u,v,\mathbf{x},\mathbf{y}_1,\mathbf{y}_2) = p(u,v)p(\mathbf{x}|u,v)p(\mathbf{y}_1,\mathbf{y}_2|\mathbf{x})$

Linearization

For any $0 \le \mu_1 \le \mu_2 \le 1$ and any $0 \le \lambda \le 1$,

$$\begin{aligned} &R_0 + \mu_1 R_1 + \mu_2 R_2 \\ &\qquad \le \lambda I(U; \mathbf{Y}_1) + (1-\lambda) I(U; \mathbf{Y}_2) + \mu_1 I(\mathbf{X}; \mathbf{Y}_1 | U, V) + \mu_2 I(V; \mathbf{Y}_2 | U) \end{aligned}$$

for some $p(u, v, \mathbf{x}, \mathbf{y}_1, \mathbf{y}_2) = p(u, v) p(\mathbf{x} | u, v) p(\mathbf{y}_1, \mathbf{y}_2 | \mathbf{x})$

Case 1: $\mu_1 \le \mu_2 \le \lambda \le 1$

- Split each receiver into two virtual receivers

- Enhance receivers that decode private messages:

$$\begin{aligned} & R_0 + \mu_1 R_1 + \mu_2 R_2 \\ & \quad \le \lambda I(U; \mathbf{Y}_1) + (1-\lambda) I(U; \mathbf{Y}_2) + \mu_1 I(X; \widetilde{\mathbf{Y}}_1 | V, U) + \mu_2 I(V; \widetilde{\mathbf{Y}}_2 | U) \end{aligned}$$

where $0 \preceq \widetilde{\mathbf{N}}_1 \preceq \widetilde{\mathbf{N}}_2 \preceq \{\mathbf{N}_1, \mathbf{N}_2\}$

- Evaluate with extremal inequality

Case 2: $\mu_1 \le \lambda \le \mu_2 \le 1$

- Split each receiver into two virtual receivers
- Enhance receiver that decodes W_1 and partially enhance receiver that decodes W_2:

$$\begin{aligned} & R_0 + \mu_1 R_1 + \mu_2 R_2 \\ & \qquad \le \ \lambda I(U; \mathbf{Y}_1) + (1-\lambda) I(U; \mathbf{Y}_2) + (\mu_2 - \lambda) I(V; \mathbf{Y}_2 | U) + \\ & \qquad\qquad \mu_1 I(X; \widetilde{\mathbf{Y}}_1 | V, U) + \lambda I(V; \widetilde{\mathbf{Y}}_2 | U) \end{aligned}$$

where $0 \preceq \widetilde{\mathbf{N}}_1 \preceq \widetilde{\mathbf{N}}_2 \preceq \{\mathbf{N}_1, \mathbf{N}_2\}$

- Evaluate with extremal inequality

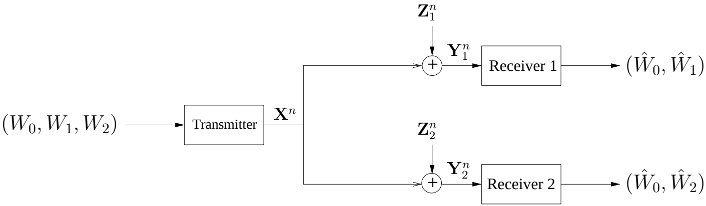
Case 3: $\lambda \leq \mu_1 \leq \mu_2 \leq 1$

- $\mu_1 + \mu_2 \leq 1 \quad \Rightarrow \quad \lambda \leq \mu_1 \leq 1 - \mu_2 \quad \Rightarrow \quad R_2 = 0$
- Problem reduced to degraded message sets

Summary

- MIMO Gaussian broadcast channel with common messages
- Maximize $R_0 + \mu_1 R_1 + \mu_2 R_2$ for $\mu_1, \mu_2 \geq 0$
- Show that the natural scheme achieves the weighted sum capacity for $\mu_1 + \mu_2 \leq 1$
- Full capacity region remains unknown

MIMO Gaussian Broadcast Channel with Common and Confidential Messages



- Secrecy constraints: $\frac{1}{n}I(W_1;\mathbf{Y}_2^n) \to 0$ and $\frac{1}{n}I(W_2;\mathbf{Y}_1^n) \to 0$
- A natural scheme that combines Gaussian superposition coding and secret dirty-paper coding achieves the entire capacity region (Liu-Liu-Poor-Shamai '10)

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